B.Sc. Part II (Hons.) 4th Paper. Diff. Equations

LDE with constant coefficients (contd.)

$$\boxed{(aD^2+bD+c)y = Q}$$

where Q is of the form $\sin px$ or $\cos px$.

$$PI = \frac{1}{f(D^2)} \sin ax$$

$$= \sin ax \times \frac{1}{f(-a^2)}$$

i.e. Put $D^2 = -a^2$ if Denominator $\neq 0$.

If it is zero then

$\sin ax$ = Imaginary part of $(\cos ax + i \sin ax)$

$\Rightarrow \sin ax$ = I.P. of e^{iax}

and $\cos ax$ = Real Part (R.P.) of e^{iax}.

1. $(D^2-9)y = \sin 2x$

Soln: For CF, $D^2-9=0$

$\Rightarrow D = \pm 3$

$\therefore CF = Ae^{3x} + Be^{-3x}$.

For PI

$$PI = \frac{1}{D^2-9}\sin 2x$$

$$\Rightarrow PI = \sin 2x \times \frac{1}{-2^2-9} = -\frac{1}{13}\sin 2x.$$

$\therefore$ complete solution is given by

$y = CF + PI$

$$\Rightarrow y = Ae^{3x} + Be^{-3x} - \frac{1}{13}\sin 2x.$$

2. $(D^2-D-2)y = \sin 2x$

Soln For CF $D^2-D-2=0$

$\Rightarrow (D-2)(D+1)=0 \quad \therefore D = 2, -1$

$\therefore CF = Ae^{2x} + Be^{-x}$

For PI,

$$PI = \frac{1}{D^2 - D - 2} \sin 2x$$

$$\Rightarrow PI = \frac{1}{-2^2 - D - 2} \sin 2x \quad \left[\text{put } D^2 = -2^2 \right]$$

$$= -\frac{1}{(D+6)} \sin 2x$$

$$= -\frac{(D-6)}{D^2 - 36} \sin 2x = -\frac{(D-6) \sin 2x}{-2^2 - 36}$$

$$= \frac{1}{40}(D-6) \sin 2x = \frac{1}{40}\left[D(\sin 2x) - 6 \sin 2x \right]$$

$$= \frac{1}{40}\left[2\cos 2x - 6 \sin 2x \right]$$

$$= \frac{1}{20}(\cos 2x - 3 \sin 2x)$$

$\therefore$ complete solution is given by

$$y = cf + PI$$

$$\Rightarrow y = Ae^{2x} + Be^{-x} + \frac{1}{20}(\cos 2x - 3 \sin 2x)$$

Q.

$$(D^3 + D^2 - D - 1)y = \cos 2x$$

Soln

For CF, $D^3 + D^2 - D - 1 = 0$

$$\Rightarrow D^2(D+1) - 1(D+1) = 0$$

$$\Rightarrow (D+1)(D^2-1) = 0 \Rightarrow (D+1)^2(D-1) = 0$$

$$\Rightarrow D = 1, -1, -1$$

$$\therefore CF = Ae^{x} + (B + Cx)e^{-x}.$$

For PI

$$y = \frac{1}{D^3 + D^2 - D - 1} \cos 2x$$

$$\Rightarrow y = \frac{1}{D^2 \cdot D + D^2 - D - 1} \cos 2x = \frac{1}{-2^2 \cdot D - 2^2 - D - 1} \cos 2x$$

$$= \frac{1}{-5D - 5} \cos 2x = \frac{1}{-5(D+1)} \cos 2x$$

$$= \frac{1}{-5(D+1)} \times \frac{D-1}{D-1} \cos 2x$$

$$= \frac{D-1}{-5(D^2-1)} \cos 2x = \frac{(D-1)\cos 2x}{-5[-2^2-1]}$$

$$= \frac{1}{25}[D(\cos 2x) - \cos 2x]$$

$$\Rightarrow PI\ y = \frac{1}{25}(-2\sin 2x - \cos 2x) = -\frac{1}{25}(2\sin 2x + \cos 2x)$$

$\therefore$ Complete soln is $y = CF + PI$

$$\Rightarrow y = Ae^{x} + (B + Cx)e^{-x} - \frac{1}{25}(2\sin 2x + \cos 2x)$$